

QUANTUM COSMOLOGY LECTURES *

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Abstract

Quantum cosmology is the quantum theory of the entire universe. Although strange at first sight, it is appropriate because (1) our world appears to be fundamentally quantum, (2) the classical description of gravity breaks down at singularities it would give at the beginning of the universe, and (3) our universe has many properties that cannot be explained by a classical description of them. A quantum state of the universe should obey certain constraints, such as the Wheeler-DeWitt equations. One approach to interpreting such a state is *Sensible Quantum Mechanics* (SQM), in which nothing is probabilistic, except, in a certain frequency sense, conscious perceptions. Sets of these perceptions can be deterministically realized with measures given by expectation values of positive-operator-valued *awareness operators*. These may be defined even when the quantum state itself is not normalizable, though there still seem to be problems of divergences when one has an infinite amount of inflation, producing infinitely large spatial volumes with presumably infinite measures of perceptions. Ratios of the measures for sets of perceptions (if finite) can be interpreted as frequency-type probabilities for many actually existing sets rather than as propensities for potentialities to be actualized, so there is nothing indeterministic in a specific SQM. One can do a Bayesian analysis to test between different specific SQMs. One can also make statistical predictions of what one might perceive within a specific SQM by invoking the *Conditional Aesthetic Principle*: among the set of all conscious perceptions, our perceptions are likely to be typical.

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1 Basics of Canonical Quantum Cosmology

Quantum cosmology is quantum mechanics applied to the whole universe. At first sight, this may seem like a strange thing to study, since quantum mechanics is supposed to apply to the very small, whereas the universe is very large. However, there are at least three motivations for studying quantum cosmology:

(1) Because it's there. Quantum mechanics certainly seems to apply to parts of the universe (e.g., microscopic systems), and for those parts, the indications are that it is more basic or fundamental than classical theory, which appears to arise as some approximation for certain macroscopic systems. Although we do not know definitely that quantum mechanics applies to the entire universe, that certainly looks like the simplest possibility consistent with our present knowledge, so it behooves us to investigate the implications if it does.

(2) Our present classical theory of gravity, Einstein's general relativity, contains within itself the seeds for its own destruction in predicting (given certain observations about the present universe and certain reasonable assumptions about the behavior of matter at high densities) that the universe was once so small and highly curved that classical theory should not have been valid. In other words, we believe the universe was once so small that indeed quantum mechanics should have been needed to understand it.

(3) Present properties of the universe can be described but not explained by classical theory: (a) the flatness of the universe, meaning its large volume and number of particles; (b) the approximate isotropy and homogeneity of the large-scale universe; (c) the particular form of the inhomogeneous structure of the universe on smaller scales; and (d) the thermodynamic arrow of time.

Since the dominant interaction on the largest scales of the universe is gravity, quantum cosmology inevitably involves quantum gravity in a fundamental way. Unfortunately, we do not yet have a complete consistent theory of quantum gravity. Superstring theory seems to be the best current candidate for becoming such a theory, but it is not yet well understood, particularly at the nonperturbative level. Part of the problem of quantum gravity is to get a theory which remains calculable (e.g., can be rendered finite) when one includes arbitrarily high energy fluctuations, and for this problem superstring theory does seem to be remarkably successful, at least at the perturbative level. But another part of the problem are more conceptual issues relating to understanding quantum mechanics for a closed system, and furthermore a system that is not simply sitting in a fixed background spacetime. These problems have not yet been satisfactorily solved, even by superstring theory, and they tend to be the focus of those of us working in quantum cosmology.

Because in certain limits superstring theory reduces to Einstein's general rela-

tivity as an approximation to it, and because many of the more conceptual issues appear to remain, and indeed stand out more clearly, when one makes the approximation to Einsteinian gravity without requiring this gravity to be classical, it is often convenient in quantum cosmology to make the approximation of taking a quantum version of Einsteinian gravity, quantum general relativity. This is not a renormalizable or finite theory, so one would run into trouble using it in multi-loop calculations that allow arbitrarily high energy fluctuations, but fortunately many of the more conceptual issues show up at a cruder approximation, often even at the WKB or ‘semiclassical’ level with a superposition of fairly classical geometries (though usually not at the complete semiclassical reduction to a single classical geometry that solves the classical Einstein equations with a unique expectation value of the stress-energy tensor as its source).

One standard method of quantizing a spatially closed universe in the approximation of Einsteinian gravity is canonical quantization. Since there exist reviews of this procedure [1, 2], I shall do nothing more here than to give a very brief sketch of the procedure. One starts with the Hamiltonian formulation [3], foliating spacetime into a temporal sequence of closed spatial hypersurfaces with lapse and shift vectors connecting them. Varying the action with respect to these Lagrange multipliers give the momentum and Hamiltonian constraint equations. Then one follows the Dirac method of quantization [4], converting these constraints into operators and requiring that they annihilate the quantum state. When the state is written as a wavefunctional of the three-metric on the spatial hypersurfaces, the momentum constraint (linear in the momenta) for each point of space implies that the wavefunctional is unchanged under any series of infinitesimal diffeomorphisms or coordinate transformations of the three-space [5, 6]. The Hamiltonian constraint (quadratic in the momenta) gives the Wheeler-DeWitt equation [7, 8], actually one equation for each point of space, or one equation for each arbitrary choice of the lapse function.

For each choice of the lapse function, one natural choice for the factoring ordering [7,9-13] of the corresponding Wheeler-DeWitt equation turns it into a Klein-Gordon equation on an indefinite DeWitt metric [7] in the superspace (space of three-metrics), with a potential term that is given by a spatial integral (weighted by the lapse function) of the spatial curvature plus the spatial gradient and potential energy terms that go into the energy density of any matter included in the model. The WKB approximation for this equation gives the Hamilton-Jacobi equation for general relativity, and the trajectories that are orthogonal to the surfaces of constant phase represent classical spacetimes that solve the classical Einstein equations.

In addition to constraint equations (the Wheeler-DeWitt equations in canonically quantized general relativity) that restrict the quantum states, a complete theory of quantum cosmology should specify which solution of these equations describes our universe. There have been various proposals for this in recent years, most notably the Hartle-Hawking ‘no-boundary’ proposal [14-16] that the wavefunctional evaluated for a compact three-geometry argument is given by a path integral over compact

‘Euclidean’ four-geometries (with positive-definite metric signatures, as opposed to ‘Lorentzian’ geometries with an indefinite signature) which reduce to the compact three-geometry argument at its boundary, and the Vilenkin tunneling proposal [17], that the wavefunctional is ingoing into superspace at its regular boundaries and outgoing at its singular boundaries.

Once one has a quantum state or wavefunctional that satisfies the constraints of canonical quantum general relativity, there is the question of how to interpret it to give probabilities. The first stage of this is to find an inner product. DeWitt [7] and many others have proposed using the Klein-Gordon inner product, using the flux of the conserved Klein-Gordon current. However, this is not positive definite and vanishes for real wavefunctionals, such as what one would get from the Hartle-Hawking ‘no-boundary’ proposal [14-16]. Another approach is to quantize the true physical variables [18], which gives unitary quantum gravity, at least at the one-loop level. However, it is not clear whether this approach can be carried beyond the perturbative level, even in a minisuperspace approximation. A third approach is third quantization, in which one converts the Wheeler-DeWitt wavefunctional to a field operator on superspace. However, I have not seen any clear way to get testable probabilities out of this.

I myself favor Hawking’s approach [16, 13] of using the naïve inner product obtained by taking the integral of the absolute square of a wavefunctional over superspace, with its volume element obtained from the DeWitt metric on it. At least this obviously gives a positive-definite inner product. Unfortunately, it will almost certainly diverge when the square of a physical wavefunctional (i.e., one satisfying the Wheeler-DeWitt equations) is integrated over all of the infinite volume of superspace (which includes directions corresponding to spatial diffeomorphisms and also to time translations). One can try to eliminate the divergences due to the noncompact diffeomorphism group by integrating only over distinct three-geometries (the older meaning of superspace) rather than over all three-metrics that overcount these, though then there is not such an obvious candidate for the volume element. However, one would still have divergences due to the noncompactness of time, which in the canonical quantum gravity of closed universes is encoded in the three-geometry and matter field configuration on the spatial hypersurface. Perhaps one can avoid these divergences by evaluating the inner product only for wavefunctionals that have been acted on by operators, such as projection operators, that do not commute with the constraints and which result in wavefunctionals that are normalizable with the naïve inner product.

2 Basics of Sensible Quantum Cosmology

Before going further with trying to calculate probabilities in quantum cosmology, it may be helpful to ask what probabilities mean in quantum mechanics. If they mean propensities for potentialities to be converted to actualities, then quantum

mechanics would seem to be incomplete, since it does not predict *which* possibilities will be actualized, but only the probabilities for each. An interpretation in which quantum mechanics would be more nearly complete would be the Everett or “many-worlds” interpretation [19], in which all possibilities with positive probabilities are actualized, though with measures that depend on the quantum probabilities. However, even this interpretation only seems to work with a single unspecified set of possibilities, whose probabilities add up to one. The arbitrariness of this set seems to indicate that the Everett interpretation is also incomplete, even though it is more nearly complete than the propensity interpretation.

Therefore, I have proposed a version of quantum mechanics, which I call Sensible Quantum Mechanics (SQM) [20-25], in which only a definite set of possibilities have measures, namely, conscious perceptions.

Sensible Quantum Mechanics is given by the following three fundamental postulates [22]:

Quantum World Axiom: The unconscious “quantum world” Q is completely described by an appropriate algebra of operators and by a suitable state σ (a positive linear functional of the operators) giving the expectation value $\langle O \rangle \equiv \sigma[O]$ of each operator O .

Conscious World Axiom: The “conscious world” M , the set of all perceptions p , has a fundamental measure $\mu(S)$ for each subset S of M .

Quantum-Consciousness Connection: The measure $\mu(S)$ for each set S of conscious perceptions is given by the expectation value of a corresponding “awareness operator” $A(S)$, a positive-operator-valued (POV) measure [26], in the state σ of the quantum world:

$$\mu(S) = \langle A(S) \rangle \equiv \sigma[A(S)]. \quad (1)$$

Here a perception p is the entirety of a single conscious experience, all that one is consciously aware of or consciously experiencing at one moment, the total “raw feel” that one has at one time, or [27] a “phenomenal perspective” or “maximal experience.”

Since all sets S of perceptions with $\mu(S) > 0$ really occur in SQM, it is completely deterministic if the quantum state and the $A(S)$ are determined: there are no random or truly probabilistic elements. Nevertheless, because SQM has measures for sets of perceptions, one can readily calculate ratios that can be interpreted as conditional probabilities. For example, one can consider the set of perceptions S_1 in which there is a conscious awareness of cosmological data and theory and a conscious belief that the visible universe is fairly accurately described by a Friedman-Robertson-Walker (FRW) model, and the set S_2 in which there is a conscious memory and interpretation of getting reliable data indicating that the Hubble constant is greater than 75 km/sec/Mpc. Then one can interpret

$$P(S_2|S_1) \equiv \mu(S_1 \cap S_2)/\mu(S_1) \quad (2)$$

as the conditional probability that the perception is in the set S_2 , given that it is

in the set S_1 , that is, that a perception included a conscious memory of measuring the Hubble constant to be greater than 75 km/sec/Mpc, given that one is aware of knowledge and belief that an FRW model is accurate.

3 Testing Sensible Quantum Cosmology Theories

The measures for observed (or, perhaps more accurately, experienced) perceptions can be used to test different SQM theories for the universe, thus grounding physics and cosmology in experience. If one had a theory in which only a small subset of the set of all possible perceptions is predicted to occur, one could simply check whether an experienced perception is in that subset. If it is not, that would be clear evidence against that theory. Unfortunately, in almost all SQM theories, almost all sets of perceptions are predicted to have a positive measure, so these theories cannot be excluded so simply. For such many-perceptions theories, the best one can hope for seems to be to find *likelihood* evidence for or against it. Even how to do this is not immediately obvious, since SQM theories merely give measures for sets of perceptions rather than the existence probabilities for any perceptions (unless the existence probabilities are considered to be unity for all existing sets of perceptions, i.e., all those with nonzero measure, but this is of little help, since almost all sets exist in this sense).

In order to test and compare SQM theories, it helps to hypothesize that the set M of all possible conscious perceptions p is a suitable topological space with a prior measure

$$\mu_0(S) = \int_S d\mu_0(p). \quad (3)$$

Then, because of the linearity of positive-valued-operator measures over sets, one can write each awareness operator as

$$A(S) = \int_S E(p) d\mu_0(p), \quad (4)$$

a generalized sum or integral of “experience operators” or “perception operators” $E(p)$ for the individual perceptions p in the set S . Similarly, one can write the measure on a set of perceptions S as

$$\mu(S) = \langle A(S) \rangle = \int_S d\mu(p) = \int_S m(p) d\mu_0(p), \quad (5)$$

in terms of a measure density $m(p)$ that is the quantum expectation value of the experience operator $E(p)$ for the same perception p :

$$m(p) = \langle E(p) \rangle \equiv \sigma[E(p)]. \quad (6)$$

Now one can test the agreement of a particular SQM theory with a conscious observation or perception p by calculating the (ordinary) *typicality* $T(p)$ that the

theory assigns to the perception: Let $S_{\leq}(p)$ be the set of perceptions p' with $m(p') \leq m(p)$. Then

$$T(p) \equiv \mu(S_{\leq}(p))/\mu(M). \quad (7)$$

For p fixed and \tilde{p} chosen randomly with the infinitesimal measure $d\mu(\tilde{p})$, the probability that $T(\tilde{p})$ is less than or equal to $T(p)$ is

$$P_T(p) \equiv P(T(\tilde{p}) \leq T(p)) = T(p). \quad (8)$$

In the case in which $m(p)$ varies continuously in such a way that $T(p)$ also varies continuously, this typicality $T(p)$ has a uniform probability distribution between 0 and 1, but if there is a nonzero measure of perceptions with the same value of $m(p)$, then $T(p)$ has discrete jumps. (In the extreme case in which $m(p)$ has one constant value over all perceptions, $T(p)$ is unity for each p .)

Thus the typicality $T_i(p)$ of a perception p is the probability in a particular SQM theory or hypothesis H_i that another random perception will have its measure density and hence typicality less than or equal to that of p itself. One can interpret it as the likelihood of the perception p in the particular theory H_i , not for p to exist, which is usually unity (interpreting all perceptions p with $m(p) > 0$ as existing), but for p to have a typicality no larger than it has.

Once the typicality $T_i(p)$ can be calculated for an experienced perception assuming the theory H_i , one approach is to use it to rule out or falsify the theory if the resulting typicality is too low. Another approach is to assign prior probabilities $P(H_i)$ to different theories (presumably neither propensities nor frequencies but rather purely subjective probabilities, perhaps one's guess for the "propensities" for God to create a universe according to the various theories), say

$$P(H_i) = 2^{-n_i}, \quad (9)$$

where n_i is the rank of H_i in order of increasing complexity (my present favorite choice for a countably infinite set of hypotheses if I could do this ranking, which is another problem I will not further consider here). Then one can use Bayes' rule to calculate the posterior probability of the theory H_i given the perception p as

$$P(H_i|p) = \frac{P(H_i)T_i(p)}{\sum_j P(H_j)T_j(p)}. \quad (10)$$

There is the potential technical problem that one might assign nonzero prior probabilities to hypotheses H_i in which the total measure $\mu(M)$ for all perceptions is *not* finite, so that the right side of Eq. (7) may have both numerator and denominator infinite, which makes the typicality $T_i(p)$ inherently ambiguous. To avoid this problem, one might use, instead of $T_i(p)$ in Eq. (10), rather

$$T_i(p; S) = \mu_i(S_{\leq}(p) \cap S)/\mu_i(S) \quad (11)$$

for some set of perceptions S containing p that has $\mu_i(S)$ finite for each hypothesis H_i . This is related to a practical limitation anyway, since one could presumably only hope to be able to compare the measure densities $m(p)$ for some small set of perceptions rather similar to one's own, though it is not clear in quantum cosmological theories that allow an infinite amount of inflation how to get a finite measure even for a small set of perceptions. Unfortunately, even if one can get a finite measure by suitably restricting the set S , this makes the resulting $P(H_i|p; S)$ depend on this chosen S as well as on the other postulated quantities such as $P(H_i)$.

Instead of using the particular typicality defined by Eq. (7) above, one could of course instead use any other property of perceptions which places them into an ordered set to define a corresponding "typicality." For example, I might be tempted to order them according to their complexity, if that could be well defined. Thinking about this alternative "typicality" leaves me surprised that my own present perception seems to be highly complicated but apparently not infinitely so. What simple complete theory could make a typical perception have a high but not infinite complexity?

However, the "typicality" defined by Eq. (7) has the merit of being defined purely from the prior and fundamental measures, with no added concepts such as complexity that would need to be defined. The necessity of being able to rank perceptions, say by their measure density, in order to calculate a typicality, is indeed one of my main motivations for postulating a prior measure given by Eq. (3).

Nevertheless, there are alternative typicalities that one can define purely from the prior and fundamental measures. For example, one might define a *reversed typicality* $T_r(p)$ in the following way (again assuming that the total measure $\mu(M)$ for all perceptions is finite): Let $S_{\geq}(p)$ be the set of perceptions p' with $m(p') \geq m(p)$. Then

$$T_r(p) \equiv \mu(S_{\geq}(p))/\mu(M). \quad (12)$$

For p fixed and \tilde{p} chosen randomly with the infinitesimal measure $d\mu(\tilde{p})$, the probability that $T_r(\tilde{p})$ is less than $T_r(p)$ is

$$P_{T_r}(p) \equiv P(T_r(\tilde{p}) \leq T_r(p)) = T_r(p), \quad (13)$$

the analogue of Eq. (8) for the ordinary typicality.

In the generic continuum case in which $m(p)$ varies continuously in such a way that there is only an infinitesimally small measure of perceptions whose $m(p)$ are infinitesimally near any fixed value, the reversed typicality $T_r(p)$ is simply one minus the ordinary typicality, i.e., $1 - T(p)$, and also has a uniform probability distribution between 0 and 1. Its use arises from the fact that just as a perception with very low ordinary typicality $T(p) \ll 1$ could be considered unusual, so a perception with an ordinary typicality too near one (and hence a reversed typicality too near zero, $T_r(p) \ll 1$) could also be considered unusual, "too good to be true."

Perhaps one might like to combine the ordinary typicality with the reversed typicality to say that a perception giving either typicality too near zero would be

evidence against the theory. For example, one might define the *dual typicality* $T_d(p)$ as the probability that a random perception \tilde{p} has the lesser of its ordinary and its reversed typicalities less than or equal to that of the perception under consideration:

$$T_d(p) \equiv P(\min [T(\tilde{p}), T_r(\tilde{p})] \leq \min [T(p), T_r(p)]) \equiv \mu(S_d(p))/\mu(M), \quad (14)$$

where $S_d(p)$ is the set of all perceptions \tilde{p} with the minimum of its ordinary and reversed typicalities less than or equal to that of the perception p , i.e., the set with $\min [T(\tilde{p}), T_r(\tilde{p})] \leq \min [T(p), T_r(p)]$. In the case in which $T(p)$, and hence also $T_r(p)$, varies continuously from 0 to 1,

$$T_d(p) = 1 - |1 - 2T(p)|. \quad (15)$$

Then the dual typicality $T_d(p)$ would be very small if the ordinary typicality $T(p)$ were very near either 0 or 1.

Of course, one could go on with an indefinitely long sequence of typicalities, say making a perception “atypical” if $T(p)$ were very near any number of particular values at or between the endpoint values 0 and 1. But these endpoint values are the only ones that seem especially relevant, and so it would seem rather *ad hoc* to define “typicalities” based on any other values. Since $T_d(p)$ is symmetrically defined in terms of both endpoints (or, more precisely, in terms of both the \leq and the \geq relations for $m(p')$ in comparison with $m(p)$), in some sense it seems the most natural one to use. Obviously, one could use it, or its modification along the lines of Eq. (11), instead of $T(p)$ in the Bayesian Eq. (10).

To illustrate how one may use these typicalities to test different theories, consider the experiment in which one makes a particular measurement of a single particular continuous variable (e.g., the position of a one-dimensional harmonic oscillator in its ground state) which is supposed to have a gaussian quantum distribution. Suppose that in the subset of perceptions S in which this measurement is believed to have been made and the result is known, there is a one-dimensional continuum of perceptions that are linearly related to the measured value of the variable, say labeled by the real number x that is the measured value of the variable, with the prior measure $d\mu_0(p) \propto dx$. (It might be more realistic to suppose that the measuring device can transmit only a discrete set of values to the brain that is doing the perceiving, but if the number of these is large, it is convenient to approximate them by a continuum. As to whether the set of perceptions is discrete or continuous, I know of no strong evidence either way. Even if the possible quantum states of the universe lay in a finite-dimensional Hilbert space, which seems doubtful but is not clearly ruled out by observational evidence, one could easily have a continuum of experience operators $E(p)$ for a continuum set of perceptions p .)

I shall furthermore make the idealized hypothesis that the measure for this subset of perceptions labeled by x is purely given by the quantum distribution of the measured variable and is not further influenced by processes in the brain. One could imagine situations in which certain measured values lead to the release of an

anaesthetic that renders the perceiver unconscious and so essentially eliminates the measure for the corresponding perceptions. At first sight such situations that grossly affect the measure for perceptions arising from a quantum measurement seemed contrived, but then I realized that noticeably unusual results of the measuring device (say results several standard deviations from the mean) could very well attract more conscious attention, over a longer time, than results that are not noticeably unusual. It seems highly plausible that the measure for a set of perceptions would increase with their alertness and with the time over which they continually occur, so noticeably unusual events that attract more attention would presumably have a higher measure than one might otherwise naïvely expect. Thus the measure for perceptions arising from the measurement of a variable with a gaussian quantum distribution could well have tails that do not decrease so fast as the original gaussian.

I shall call this effect, whereby the measure for perceptions of unusual events is increased by the attention given them, the Attention Effect. It may well be responsible for the large number of coincidences that one is aware of from anecdotal evidence. (For example, it has occurred to me several times that it was surprising for Canada and the U.S. to have ages in years that were both perfect cubes, 125 and 216 respectively, in 1992.) One can try to combat it, by, e.g., focusing on perceptions in which it is perceived that the quantum measurement was made only a second previous, say, when there would presumably not have been time for a dull result to have receded much from consciousness. So for simplicity in the following discussion I shall make the idealized assumption that the Attention Effect, as well as any other effect that distorts the measure of perceptions from what what one would calculate from the quantum measurement itself, is negligible. (I am grateful for the visit of Jane and Tim Hawking in Edmonton during my writing of the previous paragraph, which gave me the time to realize the importance of the Attention Effect. Was the timing of their arrival another coincidence?)

Assuming no Attention Effect or similar effect, the measure for x would have the gaussian distribution it inherits from the quantum measurement, say

$$m(x) \propto e^{-x^2/(2\sigma^2)}. \quad (16)$$

Within the subset of perceptions S in which this measurement is believed to have been made and the result is known, the ordinary, reversed, and dual typicalities are

$$T(x; S) = \text{erfc}(\sqrt{x^2/(2\sigma^2)}) \equiv 1 - \text{erf}(\sqrt{x^2/(2\sigma^2)}) \equiv 1 - \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{x^2/(2\sigma^2)}} e^{-z^2} dz, \quad (17)$$

$$T_r(x; S) = 1 - T(x; S) = \text{erf}(\sqrt{x^2/(2\sigma^2)}), \quad (18)$$

$$\begin{aligned} T_d(x; S) &= 1 - |1 - 2T(x; S)| = 1 - |1 - 2T_r(x; S)| \\ &= 1 - |1 - 2\text{erfc}(\sqrt{x^2/(2\sigma^2)})| = 1 - |1 - 2\text{erf}(\sqrt{x^2/(2\sigma^2)})|. \end{aligned} \quad (19)$$

Suppose that one does not know the actual standard deviation σ for x but has various hypotheses H_i that it has the various values σ_i . Then one can replace σ with σ_i in Eqs. (17)-(19) above to get the typicalities (restricted to the subset S) of the perception p that gives the perception component x in the corresponding hypothesis. If the typicality one is considering is too low for a certain hypothesis, one might use the perception to exclude (or falsify on a likelihood basis) that hypothesis.

Alternatively, one might assign a prior probability distribution to σ_i and then use the Bayesian Eq. (10), with $T_i(p)$ replaced by $T(x; S)$, $T_r(x; S)$, or $T_d(x; S)$ as given by Eqs. (17)-(19) and with σ replaced by σ_i , to calculate a posterior distribution for σ_i , given x from the perception. For example, one might assign (as a fairly simple concrete choice) the following prior probability distribution for σ_i , which is purely a decaying exponential distribution in its square and thus a slight modification of a gaussian distribution for σ_i itself:

$$P(\sigma_i) = e^{-\sigma_i^2/(2\sigma_0^2)} \sigma_i d\sigma_i / \sigma_0^2. \quad (20)$$

Here σ_0^2 is an arbitrary parameter that one must choose to represent the expectation value of σ_i^2 in the prior distribution. One can then readily calculate [28] that using the ordinary typicality in Eq. (10) gives

$$P(\sigma_i|x; S) = e^{(x/\sigma_0) - \sigma_i^2/(2\sigma_0^2)} \text{erfc}(\sqrt{x^2/(2\sigma^2)}) \sigma_i d\sigma_i / \sigma_0^2. \quad (21)$$

This is very strongly damped for small values of σ_i (i.e., for values much less than x) by the complementary error function from Eq. (17) for the typicality, and is damped at large values of σ_i (i.e., for values much greater than σ_0) by the exponentially decaying prior distribution of Eq. (20). However, if one used a different prior distribution that was not significantly damped at large values of σ_i , neither would the posterior distribution be significantly damped at large values of σ_i . Thus using the ordinary typicality T in the Bayesian Eq. (10) is effective in giving a lower limit on the spread of the measure distribution for a number like x assigned to the perceptions (at least if the one used in the calculation is not exactly at the mean value), but it is not effective in giving a better upper limit on the spread than that given by the prior distribution. This is because the ordinary typicality gives a penalty for theories that would predict that an observed result is unlikely or “bad,” but it does not give a similar penalty for theories that predict that the observed result is “too good to be likely.”

The reversed typicality T_r of Eq. (12) or (18) does indeed penalize theories that predict that the observed result is too good to be likely, but only at the cost of not penalizing at all “bad” results, so it would be worse to use. Better is the dual typicality T_d of Eq. (14) or (19), which penalizes theories that fit the observations either too poorly or too well. Unfortunately, it makes it harder to evaluate the posterior probability distribution Eq. (10) analytically, because of the minimization functions in the definition of the dual typicality, and I have been unable to come up

with a simple explicit result for the prior distribution Eq. (20), though one can get an explicit result for a prior distribution that is flat in $n \equiv 1/\sigma_i^2$ [22]. In any case, one can see that with the dual typicality, the posterior distribution for the standard deviation σ_i is more heavily damped at both large and small σ_i than is the prior distribution for σ_i that is assumed.

One can see that if one starts with a smooth continuum prior probability distribution for theories, a Bayesian analysis using one of the typicalities of an experienced perception can give a posterior probability distribution of theories that is more narrow, but it can never lead to a nonzero probability for any single theory out of the continuum of possibilities that are smoothly weighted. If this is the case, one shall never succeed in getting any single final theory that one can say has any significant (e.g., nonzero) probability of being absolutely correct. Of course, one might be able to deduce (after postulating a reasonable continuum prior distribution) fairly narrow ranges for the continuous parameters where most of the posterior probability is concentrated, so the situation could be qualitatively no worse than it is at present for such parameters as the fine structure constant, which is only thought to be likely to be within some very narrow range (and perhaps only within this range for certain components of the quantum state of the universe from which our perceptions get the bulk of their measure).

On the other hand, it is conceivable that theorists will eventually find a discrete set of theories, each with no arbitrary continuous parameters (as in superstring theories), or else with preferred discrete values of such parameters, to each of which they can assign a nonzero prior probability. Then if these are weighted by the likelihoods they predict for the perception of a sufficiently good set of observations, it is conceivable (though at present it might seem somewhat miraculous, but doesn't the order and structure in our world already look miraculous?) that the posterior probabilities will pick out one unique theory with a probability near unity and assign all other theories a total probability much closer to zero. In such a case theorists might well believe that the one unique theory with a probability near unity is indeed *the correct theory* of the universe. Of course, the fact that the other theories would not have a total probability exactly equal to zero (at least for almost any conceivable scenario I can presently imagine) would mean that one could not be sure (at least by the Bayesian analysis outlined above) that one really did have the correct theory for the universe, but if the probability were sufficiently near unity, one could presumably put a great deal of faith in that deduction from theory and observation, just as we presently put a great deal of faith in much smaller pieces of knowledge to which we assign probabilities near unity.

There is also the problem that the prior probabilities seem to be purely subjective, so people could well disagree about whether or not the assignment that led to a posterior probability near unity for one particular theory (if indeed that dream of my present wishful thinking actually does occur) is reasonable. I suspect that such disagreements about prior probabilities are at the heart of many current dis-

agreements (e.g., the existence of God or the truth of superstring theory), though there is also the huge practical problem that unless one has a detailed theory from which one can make the relevant calculations, one cannot even predict what the likelihoods are for the various hypotheses to result in certain observations (e.g., the perception of good and evil or the experience of gravity). However, one could imagine a society which agrees in sufficient broad outline about the prior probabilities, and observational results that sufficiently narrow them down (as often occurs to a fantastic degree in experimental physics, viz. the amazing agreement of experiment with QED), that nearly all members will agree on a unique theory for the universe as having a posterior probability near unity (similar to the fact that most members of the community of physicists believes in QED within its domain of applicability, though QED represents a continuum of theories labeled by the fine structure constant rather than one unique theory).

One can also worry that if Sensible Quantum Mechanics is correct, then presumably our perceptions are completely determined by the detailed theory, and it seems likely that it would only be some sort of an idealization to say that our beliefs are determined by a Bayesian process of modifying prior probabilities to posterior probabilities in the light of the likelihoods of our observations. So it may be purely hypothetical to predict what beliefs we would arrive at if we went through this ideal Bayesian procedure, since it would seem miraculous if we were determined to act in just that way, and there is certainly evidence that most of the time our beliefs are not determined precisely thus. However, idealizations are at the heart of physics, and this Bayesian one does not seem particularly worse than many others. Even if the physics community (or the broader human community) does not actually come to its beliefs by precisely a Bayesian analysis, it is interesting to speculate on what conclusions it might eventually arrive at if it did. Though even such conclusions are not guaranteed to be true (because of the difficulties mentioned above, and various others), it would seem that they would be more likely to be true than whatever conclusions people will actually come up with, especially if they choose to ignore the Bayesian procedure.

Thus, although Sensible Quantum Mechanics is my current best guess of what is a true framework for a complete description of the universe, my advocacy of a Bayesian analysis to choose between detailed theories within that framework is not a guess of what is truly the way physicists work, but a moral appeal for one way in which I think the search for a detailed theory ought to be conducted.

4 Predictions from Sensible Quantum Cosmology

Although it is certainly an open question whether humans or any other conscious beings within the universe will ever come to some sort of a grasp of a complete theory of the universe (perhaps only as a set of ideas whose logical implications include a complete description of the entire universe, even though it seems extremely unlikely

that conscious beings within the universe could ever work out all of these implications in detail), we would like to develop better theories that will enable us to predict more properties of the universe than we can at present.

Vilenkin [29] has recently discussed predictions in quantum cosmology, and I do not have much in detail to add to what he beautifully covered. I agree with him that if the ‘constants’ of physics that we have measured can actually vary from component to component of the quantum state of the universe, the relevant probability distribution must be obtained by using something like the *Principle of Mediocrity* that he proposes, that we are a ‘typical’ civilization within the ensemble of components.

I might personally prefer [22, 24] a slightly different variant of the Weak Anthropic Principle [30-36] which I call the *Conditional Aesthetic Principle*, that our conscious perceptions are likely to be typical perceptions in the conscious world with its measure. Then the relevant probability distribution for the ‘constants’ of physics would be weighted by the measures for perceptions. The most basic way to do this would be to use only the perceptions which include a belief in the value of the corresponding ‘constant.’ However, if the quantum state of the universe is represented by the density matrix ρ , and if the ‘constants’ are indeed constant over the relative density matrix [22, 24]

$$\rho_p = \frac{E(p)\rho E(p)}{\text{Tr}[E(p)\rho E(p)]} \quad (22)$$

for each perception p , then it might be better to weight the probability distribution of the ‘constants’ in each such relative state by the measure density for the corresponding perception. One might like thus to find out the probability distribution for the cosmological constant, the parameters of the Standard Model of particle physics, and the parameters of an inflaton potential (if any such exists).

One persistent problem that hampers predictions even from simple toy minisuperspace models in quantum cosmology is the apparent lack of normalizability of most of the quantum states, at least if one takes the positive-definite naïve inner product obtained by integrating the absolute square of the wavefunctional over superspace. As discussed above, part of this problem in canonical quantum cosmology is due to the fact that the wavefunction in the configuration representation is most easily handled as a functional of three-metrics that is invariant under coordinate transformations, but this diffeomorphism group is noncompact and leads to divergences when one integrates over it. It seems that this ought to be merely an avoidable technical problem that arises from writing down wavefunctionals of three-metrics rather than of three-geometries, but even if one circumvents it (or avoids it by truncating the configuration space, as in homogeneous minisuperspace models), one next faces the problem of the invariance under the transformations generated by the Wheeler-DeWitt equations, which represent time translations at each point of space. It would seem that then one must face at least the noncompact groups

of Lorentz boosts at each point of space, since the hypersurface described by the three-metric argument of the wavefunctional can be tilted rather arbitrarily at each point of space. Even if those divergences are eliminated (as in the minisuperspace models in which the homogeneous three-geometries are prevented from being tilted), one can still get divergences from the infinite range of values allowed by the ‘time.’

Perhaps these divergences can be avoided by not seeking to interpret the integral of the absolute square of the physical wavefunctional over the configuration space, but by restricting the interpretation, as in Sensible Quantum Mechanics, to the integrals of wavefunctionals that have been operated on by the experience or perception operators $E(p)$. There seems to be no reason why the resulting wavefunctionals should be physical wavefunctionals that obey the constraints such as the Wheeler-DeWitt equation, since a perception could very well be localized to be concentrated near a particular clock time (represented by some property of the three-geometry or matter fields on it) and need not occur over the whole history (or, more accurately, set of histories in the path integral sense) of the universe that is represented by a physical solution of the constraints. Certainly in minisuperspace one can get positive operators, as the experience operators are required to be, that have finite expectation values even for wavefunctionals that are not normalizable, e.g., projection operators to finite ranges of all the configuration space variables.

However, I am still not sure that even this procedure will avoid all divergences. If one has a model in which an infinite amount of inflation can occur, which would be useful for solving the flatness problem [13], then in the resulting infinite amount of spatial volume it would seem that the measure for almost any set of perceptions of nonzero prior measure would likely be infinite, since each sufficiently large finite volume would seem likely to give an independent positive contribution to it. If one has infinite measures for almost all nontrivial sets of perceptions, then calculating conditional probabilities and typicalities by taking the ratios of such infinite quantities is meaningless, and I do not yet see how to get testable predictions out from even Sensible Quantum Cosmology.

In conclusion, Sensible Quantum Mechanics seems to give an improved interpretation of quantum mechanics that is helpful in quantum cosmology for testing different quantum theories of the universe and in making predictions. It seems to ameliorate part of the problem of the lack of normalizability of the wavefunctionals of canonical quantum gravity, but serious problems still seem to remain with this that hamper predictions from quantum cosmology.

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